

# **CRACK IDENTIFICATION IN BEAMS**

*A thesis submitted in the partial fulfilment of the requirements for the  
Degree of*

**BACHELOR OF TECHNOLOGY**

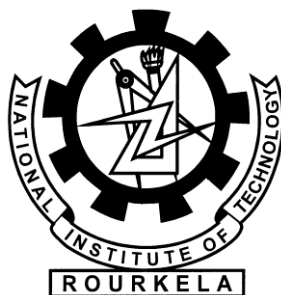
**IN**

**CIVIL ENGINEERING**

**BY**

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**UNDER THE GUIDANCE OF  
UTTAM KUMAR MISHRA**



**DEAPARTMENT OF CIVIL ENGINEERING**

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**NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA  
2013**

**CERTIFICATE**

This is to certify that the Project Report entitled, “**CRACK IDENTIFICATION IN BEAMS**” submitted by **AKASH ROUT (Roll-109CE0456)** and **RALLAPALLI SRIVASTAV (Roll-109CE0548)** in partial fulfilment for the requirements for the award of the Degree of Bachelor of Technology in Civil Engineering at National Institute of Technology, Rourkela (Deemed university) is an authentic work carried out by them under my supervision and guidance. To the best of my knowledge, the matters embodied in the thesis have not been submitted to any other university/Institute for the award of any Degree or Diploma.

Place: NIT Rourkela  
Date: 10<sup>th</sup> May 2013

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## **Abstract**

Crack in any structure changes the dynamic behaviour of the structure and by examining this change location and severity of the crack can be identified. Non-destructive testing (NDT) methods are used for detecting the location and severity i.e. crack size but these techniques are costly and time consuming. Modal parameters like natural frequency, mode shape can be used to detect the crack in beams. The present work is aimed for detection of open transverse crack in a Euler Bernoulli beam. The crack considered is an open crack and the analysis is made for linear behaviour of the beam.

Finite element method is adopted for the dynamic analysis of the beam. Additional flexibility coefficients of the cracked beam element are computed using 6-point Gaussian quadrature and theories of fracture mechanics. The total flexibility matrix is obtained by adding the additional flexibility coefficients to the intact element flexibility matrix. Then from the total flexibility matrix, overall flexibility matrix is obtained. Stiffness matrix of the cracked element is derived from the overall flexibility matrix of the element for the analysis of an Euler Bernoulli beam. The first four natural frequencies and the corresponding mode shapes of vibration are obtained by dynamic analysis solving the Eigen value problem using a FORTRAN code. These natural frequencies are used for the crack detection. 3D graphs of normalized frequency (cracked beam frequency/intact beam frequency) in terms of crack depth and crack location are plotted. The intersection of these contours gives the crack depth and crack location.

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## NOMENCLATURE

A Cross-section area of the element

b Width of the element

h ,d depth of the element

E Young's modules

G Modules of rigidity

$G_e$  Element geometric matrix

I Moment of inertia

a Crack depth

$[K_b]$  Stiffness due to bending

$[K_e]$  Element stiffness matrix

$[K_c]$  Crack beam element stiffness matrix

$[K_g]$  Geometric matrix

$[M_e]$  Element mass matrix

$[M]$  Mass matrix

L Length of the beam

$L_e$  Length of the beam element

P Applied load

m mass per unit length

$\alpha$  static load factor

$\beta$  dynamic load factor

$\rho$  mass density

RCD Relative crack depth

L1 Distance of crack from free end



# CHAPTER-1

## INTRODUCTION

## 1.1 INTRODUCTION

Civil structures in its lifetime are subjected to various dynamic loads like earthquake load, seismic load etc. which may act separate or in combination of these loads and hence an early detection of cracks are very important as these may lead to catastrophic failure leading to heavy loss of life and property. Crack identification methods are mainly based on changes in natural frequency or mode shapes. NDT methods are once used for the crack detection but these methods requires the location of damage before using these techniques and the damage part should be accessible which makes the work very time consuming in case of pipelines and long beams. These drawbacks have led to the development of global vibration based damage detection methods. In crack some materials are removed during the loading which leads to decrease in stiffness and increase in damping and a reduction in the natural frequency and these shifts are used for locating the crack and its severity.

The use of global vibration based damage detection methods instead of Non-destructive testing is due to the fact that natural frequency of a beam can be measured from any location on the beam offering scope for the development of a fast and global Non-destructive evaluation technique. These have led to considerable saving in time, labour and cost making it very effective. In this study Euler Bernoulli beam have been used with both ends free. The crack assumed is a transverse and open. All the numerical analysis of the beam has been done with suitable numerical models with the help of the computer programme.

# **C**CHAPTER-2

## **LITERATURE REVIEW**

## 2.1 LITERATURE REVIEW

J.K.SINHA et al. [1] (2002) have developed a simplified approach to model cracks in beams undergoing transverse vibration which uses Euler-Bernoulli beam elements with modifications of flexibility near the cracks and this developed model was in turn used to estimate the crack size and location

H.NAHVI et al. [2] (2005) have proposed an approach to identify crack location and depth in a cantilever open cracked based on measured frequencies and mode shapes of the beam. The crack is identified by plotting contours of normalised frequency with normalised crack depth and location and by finding the intersection of contours with constant modal frequency planes.

CHAUDHARI et al. [3] have modelled the crack in the beam of constant thickness and linear varying depth as a rotational spring and used Frobenius method to detect the crack location and depth based on measured natural frequencies.

LEE et al. [4] have presented boundary element method for solving the natural frequencies of cracked beam and the inverse problem of finding crack location and depth from natural frequencies measured has been solved using Newton-Raphson method with experimentally measured frequencies as inputs.

In the present analysis, it was taken in account that strain energy of the cracked beam is increased due to presence of crack. The additional element flexibility matrix was obtained and added intact element flexibility matrix to get total flexibility matrix of element. Then the stiffness matrix of a cracked element is calculated and the global stiffness matrix is developed, consistent mass matrix is also developed so that eigen value can be solved to find the modal frequencies of cracked beam. 3D graphs of normalized frequency (cracked beam

frequency/intact beam frequency) in terms of crack depth and crack location are plotted. The intersection of these contours gives the crack depth and crack location.

# CHAPTER-3

## THEORY AND FORMULATION

### 3.1 Dynamic stability studies

For a doubly curved panel under in-plane load, the equation of motion can in matrix form is as follows:

$$[M]\{\ddot{q}\} + [K_b] - P[K_g] \{q\} = 0 \quad (1)$$

In-plane load  $P(t)$  can be expressed in the form as shown below

$$P(t) = P_s + P_t \cos \Omega t \quad (2)$$

Where  $P_s$  is the static portion of  $P$ .  $P_t$  is the amplitude of the dynamic portion of  $P$  and  $\Omega$  is the frequency of excitation.

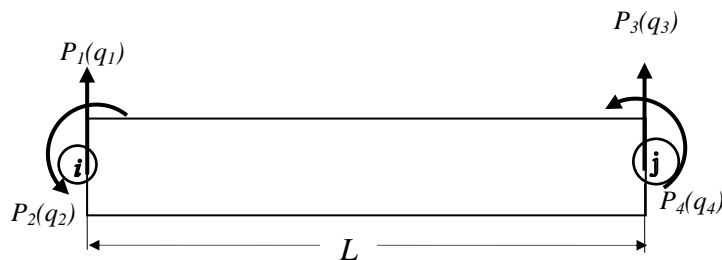
For free vibration analysis  $P(t)=0$ , so the equation can written as

$$[M]\{\ddot{q}\} + [K_b] \{q\} = 0 \quad (3)$$

This equation is inturn a eigen value problem and by solving this equation the eigen values obtained are square of natural frequency

### 3.2 FINITE ELEMENT ANALYSIS

In this analysis two noded beam elements with two degree of freedom (slope and deflection) per node is considered.



**Fig. 3.1 Two noded beam elements with two degree of freedom (deflection and slope)**

The displacement model taken as the polynomial as

$$q = a_1 + a_2x + a_3x^2 + a_4x^3 \quad (4)$$

From the displacement model and putting the boundary conditions we arrive

$$1) \text{ At } x = 0, q = q_1 \text{ so } q = a_1$$

$$\text{At } x = 0, q' = \theta_1 \text{ so } \theta_1 = a_2 \quad (5.a)$$

$$2) \text{ At } x = l, q = q_2 \text{ and } q' = \theta_2$$

$$q_2 = q_1 + \theta_1 l + a_3 l^2 + a_4 l^3 \quad (5.b)$$

$$\theta_2 = \theta_1 + 2 a_3 l + 3 a_4 l^2 \quad (5.c)$$

From equation (5) we get

$$a_1 = q_1 \quad (6.a)$$

$$a_2 = \theta_1 \quad (6.b)$$

$$a_3 = -\frac{3}{l^2} q_1 - \frac{2}{l} \theta_1 + \frac{3}{l^2} w_2 - \frac{\theta_2}{l} \quad (6.c)$$

$$a_4 = \frac{2}{l^3} w_1 + \frac{\theta_1}{l_2} - \frac{2}{l_3} w_2 + \frac{\theta_2}{l_2} \quad (6.d)$$

Writing in matrix form

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{l^2} & -\frac{2}{l} & \frac{3}{l^2} & -\frac{1}{l} \\ \frac{2}{l^3} & \frac{1}{l^2} & -\frac{2}{l^3} & \frac{1}{l^2} \end{bmatrix} \quad (7)$$

From Equation (4) we know  $q = a_1 + a_2x + a_3x^2 + a_4x^3$

$$\text{So } q = [1 \quad x \quad x^2 \quad x^3] \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} \quad (7)$$



$$q = [1 \quad x \quad x^2 \quad x^3] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{l^2} & -\frac{2}{l} & \frac{3}{l^2} & -\frac{1}{l} \\ \frac{2}{l^3} & \frac{1}{l^2} & -\frac{2}{l^3} & \frac{1}{l^2} \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} \quad (8)$$

$$\text{So } q = [N] \{q\} \quad (9)$$

$$\text{Where } \{q\} = \begin{pmatrix} q_1 \\ \theta_1 \\ q_2 \\ \theta_2 \end{pmatrix}$$

$$N = [N_1 \quad N_2 \quad N_3 \quad N_4] \quad (10)$$

$$\text{Where } N_1 = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \quad (11.a)$$

$$N_2 = x - \frac{2x^2}{l} + \frac{x^3}{l^2} \quad (11.b)$$

$$N_3 = \frac{3x^2}{l^2} - \frac{2x^3}{l^3} \quad (11.c)$$

$$N_4 = -\frac{x^2}{l} + \frac{x^3}{l^2} \quad (11.d)$$

$$[B] = \frac{\partial}{\partial x^2} [N] \quad (12)$$

Where  $[B]$  = strain displacement matrix

$$[B] = [B_1 \quad B_2 \quad B_3 \quad B_4] \quad (13)$$

Following standard procedures the stiffness matrix, mass matrix and geometric matrix can be expressed as follows

Element stiffness matrix due to bending

$$[K_e] = \int_0^L [B]^T [D] [B] dx$$

$$[K_e] = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \quad (14)$$

Element mass matrix

$$[M_e] = \int_0^l [N]^T \rho [N] dx$$

$$M_e = \frac{ml}{630} \begin{bmatrix} 234 & 33l & 81 & -19.5l \\ 33l & 6l^2 & -19.5l & -4.5l^2 \\ 81 & -19.5l & 234 & -33l \\ -19.5l & -4.5l^2 & -33l & 6l^2 \end{bmatrix} = \frac{\rho AL}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 4L^2 & 3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \quad (15)$$

Element geometric matrix

$$[K_g] = \int_0^l [N']^T [N'] dx$$

$$G_e = \frac{P}{30} \begin{bmatrix} \frac{36}{L} & 3 & \frac{-36}{L} & 3 \\ 3 & 4L & -3 & -L \\ \frac{-36}{L} & -3 & \frac{36}{L} & -3 \\ 3 & -L & -3 & 4L \end{bmatrix} \quad (16)$$

Where A= cross-sectional area of the element

$\rho$  = Mass density of the beam

### 3.3 Stiffness matrix for cracked beam element

The stiffness matrix for a cracked beam element is obtained by first calculating the total flexibility matrix and finding the inverse of it. The total flexibility matrix is sum of the

intact flexibility matrix and additional flexibility matrix due to existence of crack which is due to energy release and additional deformation of structure occurs.

**a) Elements of the overall additional flexibility matrix  $C_{oval}$**

Let  $b$  = breadth of the beam element

$h$  = depth of the cracked beam element

$L_c$  = Distance between the right hand side end node  $j$  and crack location

$L_e$  = Length of the beam element

$A$  = Cross-sectional area of the beam

$I$  = Moment of inertia

According to Diamarogonas *et al* [5], the additional strain energy existence of crack

can be expressed as  $\Pi_c = \int_A G dAc$  (17)

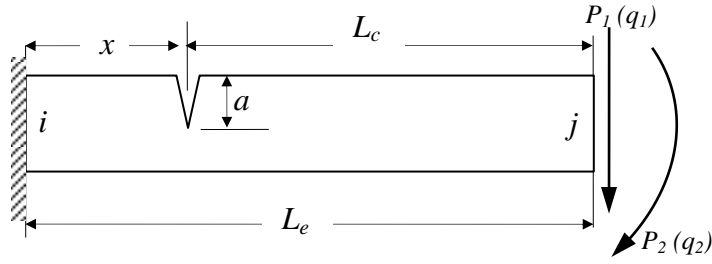
Where,  $G$  = the strain energy release rate

$Ac$  = the effective cracked area.

$$G = \frac{1}{E'} [(\sum_{n=1}^2 K_{In})^2 + (\sum_{n=1}^2 K_{II})^2 + k(\sum_{n=1}^2 K_{III})^2] \quad (18)$$

Where,  $E' = E$  for plane stress

$= E/(1-\nu^2)$  for plane strain case  $k=1+\nu$  (19)



**Figure 3.2 A typical cracked beam element subjected to shearing force and bending moment**

$K_I$ ,  $K_{II}$ , and  $K_{III}$  = Stress intensity factors for opening, sliding and tearing type cracks respectively

Neglecting the effect of axial force and for open crack the eqn. becomes

$$G = 1/E' [(K_{II} + K_{I2})^2 + K_{III}^2] \quad (20)$$

The expression for the stress intensity factors from earlier studies is given by,

$$K_{II} = \frac{6p_1 L_c}{bh^2} \sqrt{\pi \xi F_1} \left( \frac{\xi}{h} \right) \quad (21.a)$$

$$K_{I2} = \frac{6P_2}{bh^2} \sqrt{\pi \xi F_1} \left( \frac{\xi}{h} \right) \quad (21.b)$$

$$K_{III} = \frac{P_2}{bh} \sqrt{\pi \xi F_2} \left( \frac{\xi}{h} \right) \quad \text{where } s = \left( \frac{\xi}{h} \right) \quad (21.c)$$

$$\text{Where } F_1(s) = \sqrt{\frac{\tan\left(\frac{\pi s}{2}\right)}{\left(\frac{\pi s}{2}\right)}} [0.923 + 0.199(1 - \sin(\pi s/2))^4 / \cos(\pi s/2)] \quad (22)$$

When  $x = \xi/h$

$$F_{II}(x) = 1.122 - 0.561s + 0.085s^2 + 0.180s^3 / \sqrt{1-s}. \quad (23)$$

$F_I(s)$  and  $F_{II}(s)$  are the correction factors. From definition, the elements of the overall additional stiffness matrixes  $C_{ij}$  can be expressed as

$$C_{11} = \frac{2\pi}{E'bh} \left[ \frac{36L_c^2}{h^2} \int_0^{\frac{a}{h}} x F_I^2(x) dx + \int_0^{\frac{a}{h}} F_{II}^2(x) dx \right] \quad (24.a)$$

$$C_{12} = \frac{72\pi L_c}{E'bh^2} \left[ \int_0^{\frac{a}{h}} x F_I^2(x) dx \right] = C_{21} \quad (24.b)$$

$$C_{22} = \frac{72\pi}{E'bh^2} \left[ \int_0^{\frac{a}{h}} x F_I^2(x) dx \right] \quad (24.c)$$

Now the overall stiffness matrix  $C_{ovl}$  is given by

$$C_{ovl} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \quad (25)$$

**b) Flexibility matrix  $C_{intact}$  of the beam element**

$$C_{intact} = \begin{bmatrix} \frac{L_e^3}{3EI} & \frac{L_e^2}{2EI} \\ \frac{L_e^2}{2EI} & \frac{L_e}{EI} \end{bmatrix} \quad (26)$$

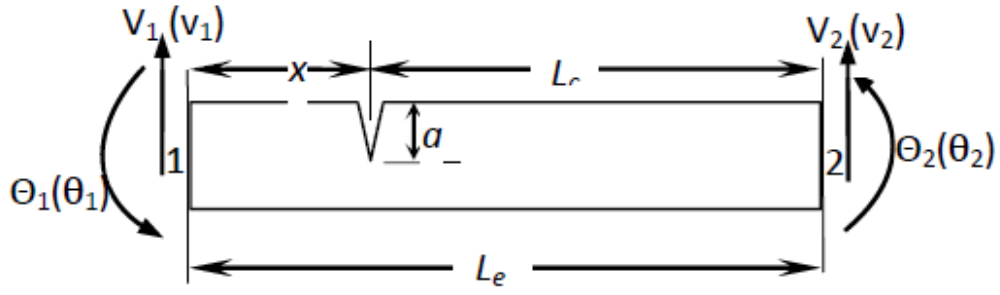
**c) Total flexibility matrix  $C_{tot}$  of the cracked beam element**

$$C_{total} = C_{intact} + C_{ovl} \quad (27)$$

**d) Stiffness matrix  $K_C$  of a cracked beam element:**

From equilibrium condition as in fig. 2

$$(V_1 \theta_1 \ V_2 \ \theta_2)^T = [L] (V_2 \ \theta_2)^T$$



**Figure3.3 Typical Cracked beam element subject to shearing force and bending moment**

$$L = \begin{bmatrix} -1 & 0 \\ L_e & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (28)$$

Hence the stiffness matrix  $K_C$  of a cracked beam element can be obtained as

$$K_{crack} = LC_{tot}^{-1}L^T \quad (29)$$

# CHAPTER-4

## RESULT AND DISCUSSION

## 4.1 COMPARISON STUDY

For comparing the results with Sinha et al [1] (2002) an aluminium beam was taken with the following properties:

- Length of the beam = 1832mm
- Width of the beam = 50mm
- Depth of the beam = 25mm
- Young's Modulus of the beam = 69.79GPa
- Density of the beam = 2600Kg/m<sup>3</sup>
- Poisson's ratio = 0.33
- No of elements = 16
- Boundary conditions of the beam = Free-Free
- DOF at each node=2(rotation & translation)
- Element Length =  $\frac{1.832}{16} = 0.1145\text{m}$
- Single open crack at a distance of 275mm and varying depths of 8 & 12mm.

The natural frequencies for the intact and cracked beam for various crack locations and crack depths were found out using the FORTRAN code and were tabulated. These frequencies are then divided by the intact beam frequencies to get required normalised frequencies. Normalised frequencies are then used to plot contours for different modes. Experimental normalised frequency is calculated. Contours corresponding to this normalised frequency are retrieved using MINITAB 16 software. Intersection of these normalised frequency contours gives the location and depth of the crack.



**Table 4.1 Normalized frequency for varying crack depths and locations for first mode**

L1/L	RCD=0.0	RCD=0.1	RCD=0.3	RCD=0.5	RCD=0.7
0.1	1	0.999302	0.99307	0.977385	0.939285
0.2	1	1.000081	0.999865	0.999109	0.996642
0.3	1	0.999857	0.998207	0.993657	0.979471
0.4	1	0.999244	0.993302	0.977162	0.928987
0.6	1	0.999244	0.993301	0.97717	0.92903
0.7	1	0.999864	0.998212	0.993536	0.978788

**Table 4.2 Normalized frequency for varying crack depths and locations for second mode**

L1/L	RCD=0.0	RCD=0.1	RCD=0.3	RCD=0.5	RCD=0.7
0.1	1	1.000196	0.999064	0.996139	0.989145
0.2	1	0.999652	0.996144	0.986484	0.95792
0.3	1	1.000209	0.989182	0.964287	0.898516
0.4	1	0.999441	0.994442	0.981281	0.945923
0.6	1	0.999438	0.99444	0.981331	0.946184
0.7	1	0.99875	0.989183	0.964274	0.898469

**Table 4.3 Normalized frequency for varying crack depths and locations for third mode**

L1/L	RCD=0.0	RCD=0.1	RCD=0.3	RCD=0.5	RCD=0.7
0.1	1	1.000554	1.000492	1.000089	0.998559
0.2	1	0.998814	0.989856	0.966996	0.910656
0.3	1	0.999698	0.99632	0.987908	0.968117
0.4	1	0.999978	0.997692	0.99142	0.974036
0.6	1	0.999969	0.997686	0.991567	0.974838
0.7	1	0.999712	0.99633	0.98767	0.966896

**Table 4.4 Normalized frequency for varying crack depths and locations for fourth mode**

L1/L	RCD=0.0	RCD=0.1	RCD=0.3	RCD=0.5	RCD=0.7
0.1	1	1.000442	0.998731	0.994027	0.981243
0.2	1	0.999246	0.991773	0.973967	0.937484
0.3	1	1.000426	0.999305	0.996319	0.988261
0.4	1	0.99872	0.988934	0.965137	0.912667
0.6	1	0.998719	0.988933	0.96516	0.912831
0.7	1	1.000481	0.999345	0.99531	0.982238

### 3-D PLOT OF NORMALIZED FREQUENCY

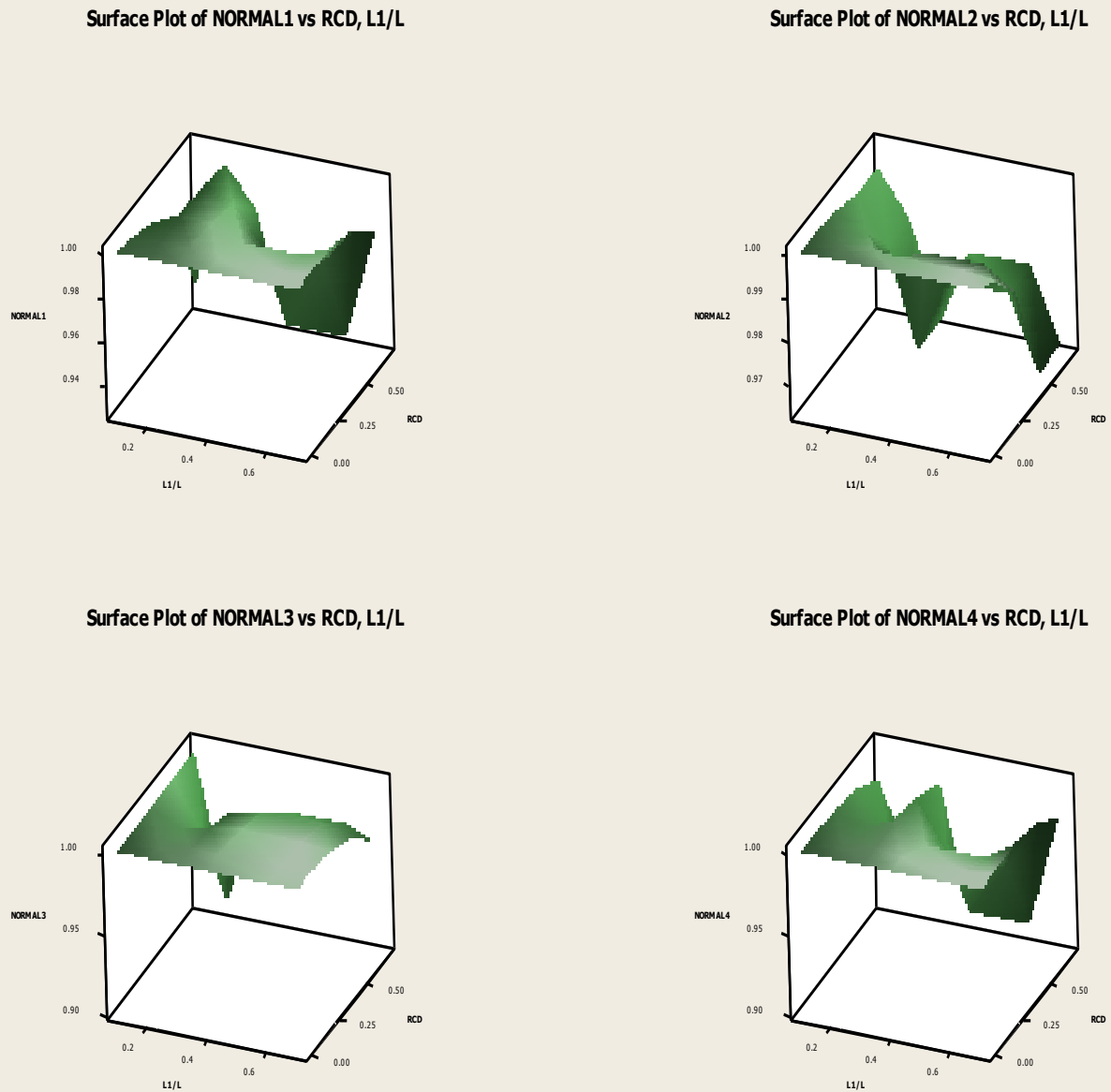


Figure 4.1 Surface plot of normalised modal frequencies

## NORMALIZED FREQUENCY CONTOURS PLOTTED

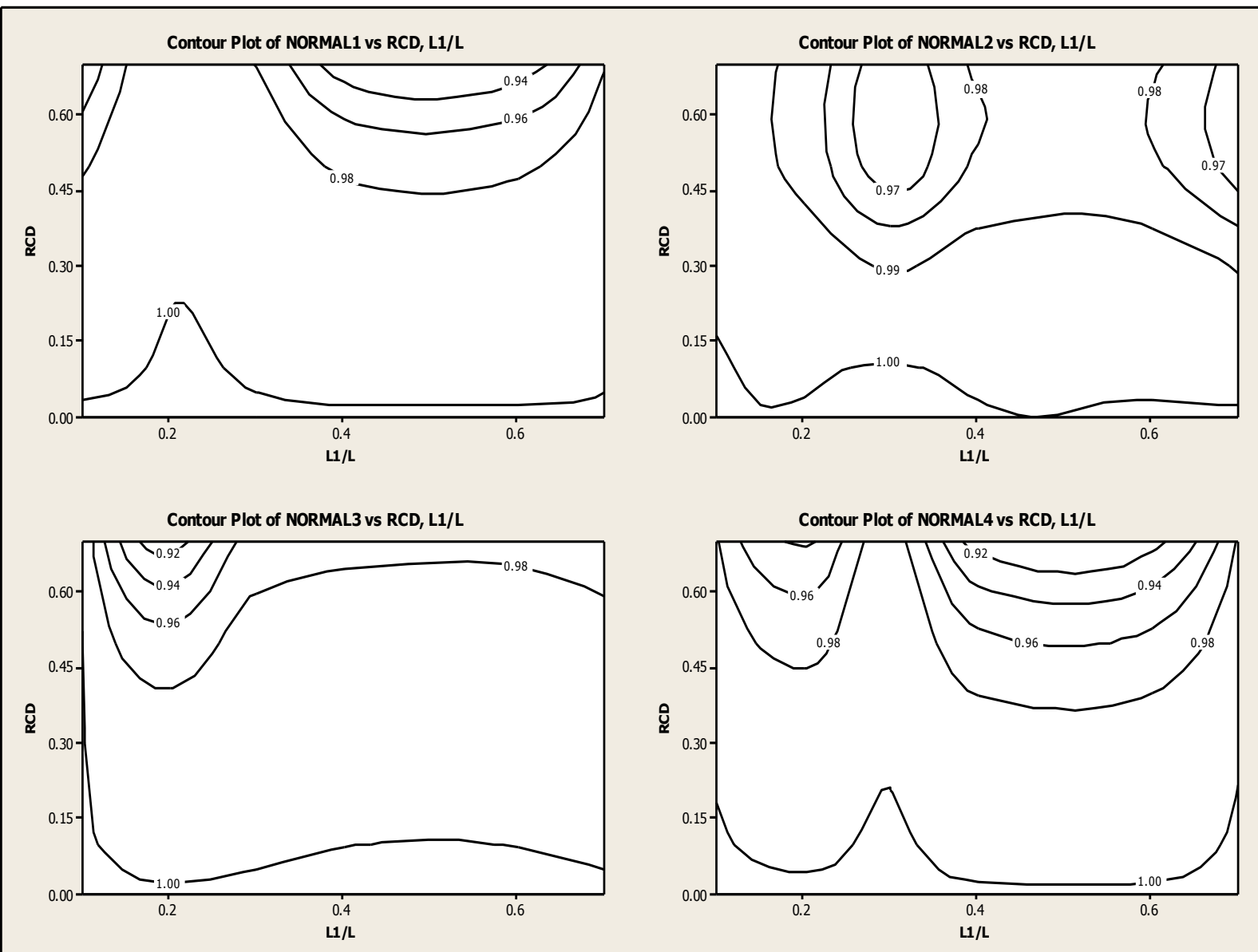


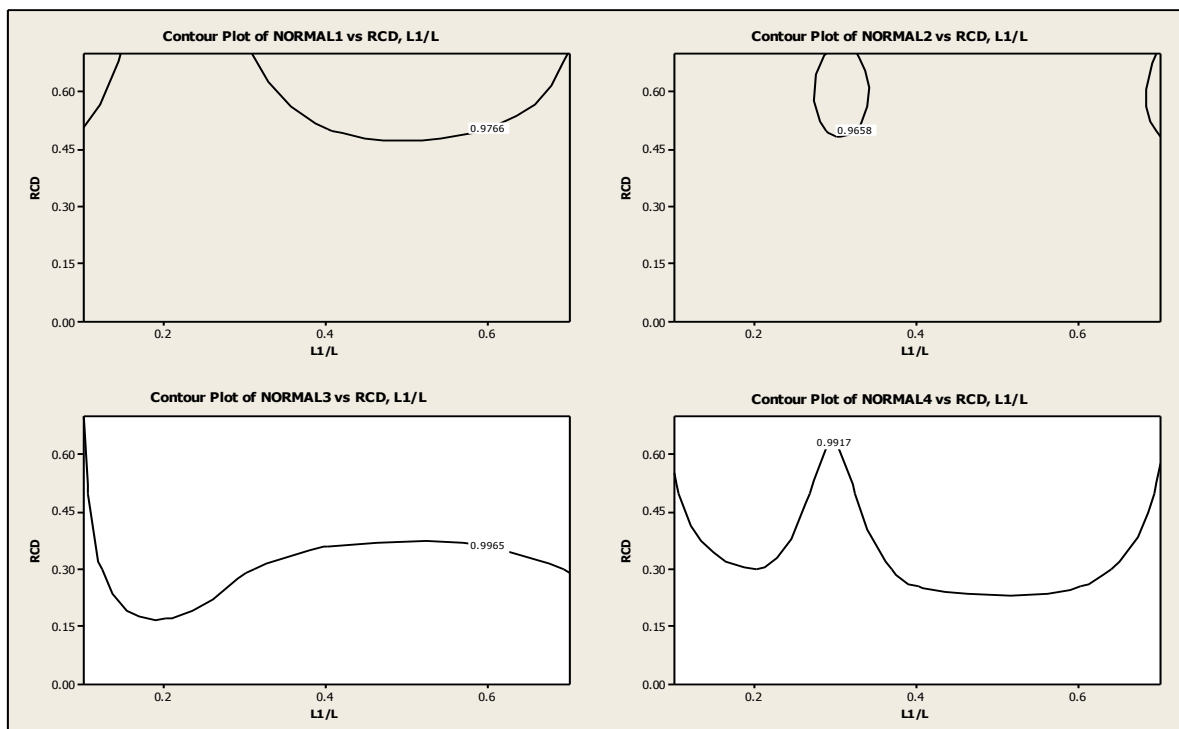
Figure 4.2 Contour plots of noramlised modal frequencies

**Table 4.5 Experimental frequency (Hz) results of intact and cracked beam (from  
SINHA et al. [1] (2002))**

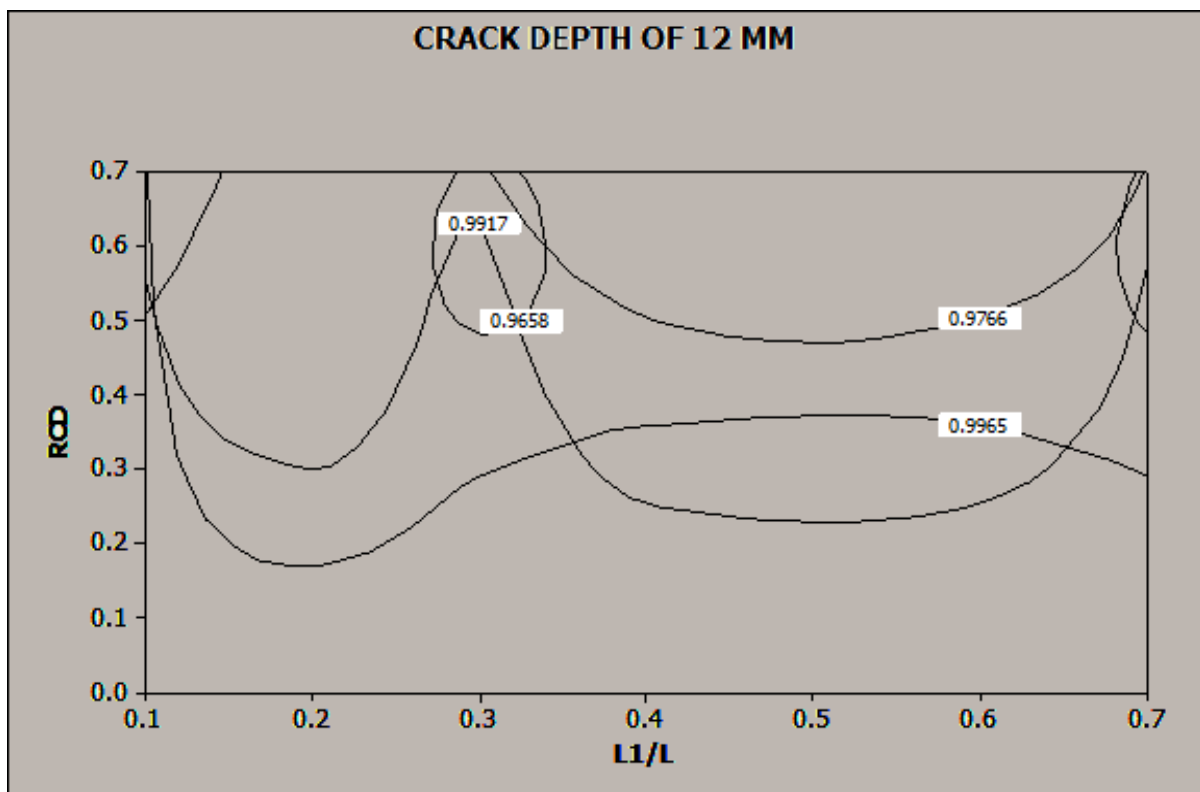
<b>CRACK</b>	<b>NO</b>	<b>8mm crack at distance of</b>	<b>12mm crack at distance of</b>
	<b>CRACK</b>	<b>275mm</b>	<b>275mm</b>
<b>MODE1</b>	<b>40</b>	<b>39.375</b>	<b>39.063</b>
<b>MODE2</b>	<b>109.688</b>	<b>108.125</b>	<b>105.938</b>
<b>MODE3</b>	<b>215</b>	<b>214.688</b>	<b>214.375</b>
<b>MODE4</b>	<b>355</b>	<b>353.438</b>	<b>350.625</b>

**Table 4.6 Experimental normalised frequency**

<b>CRACK</b>	<b>8mm crack at distance of</b>	<b>12mm crack at distance of</b>
	<b>275mm</b>	<b>275mm</b>
MODE1	0.9844	0.9766
MODE2	0.9858	0.9658
MODE3	0.9981	0.9965
MODE4	0.9956	0.9917



**Figure 4.3 12 mm crack normalised frequency retrieved**



**Figure 4.4 Crack depth of 12mm at a distance of 275mm**

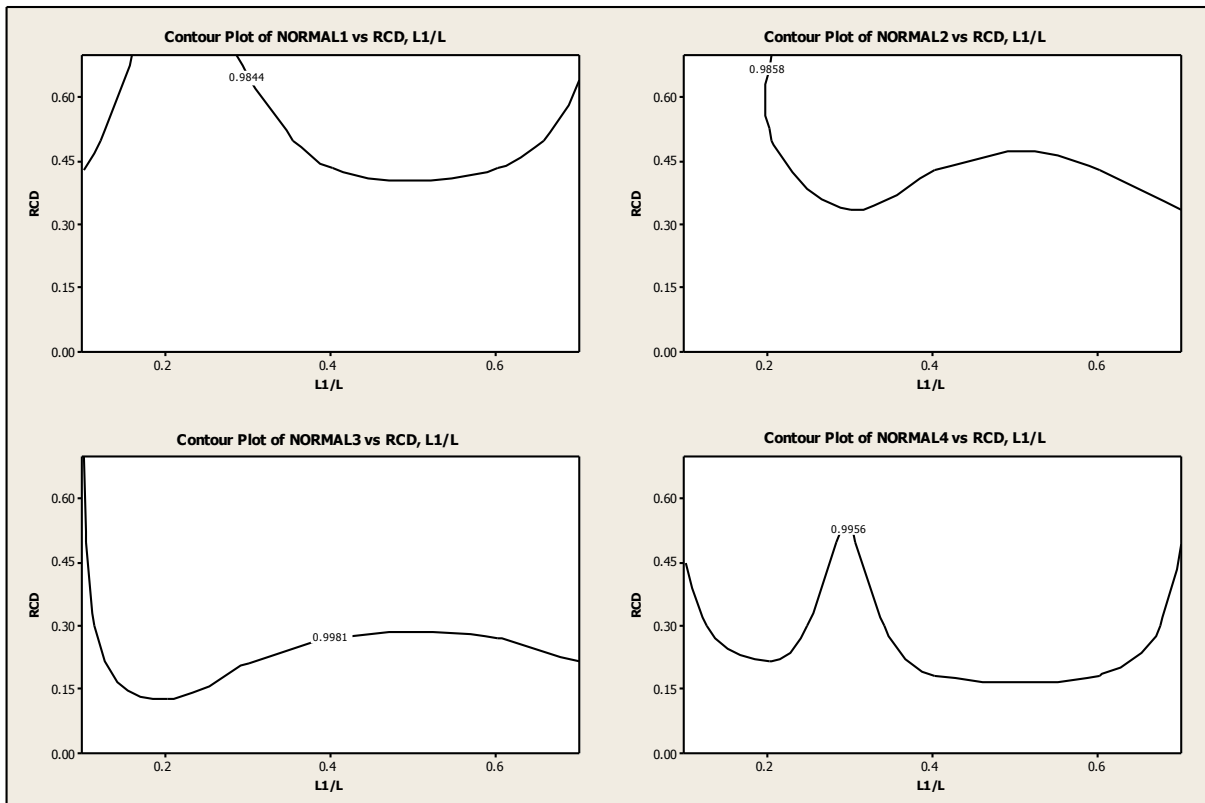


Figure 4.5 8 mm Crack normalised frequency retrieved

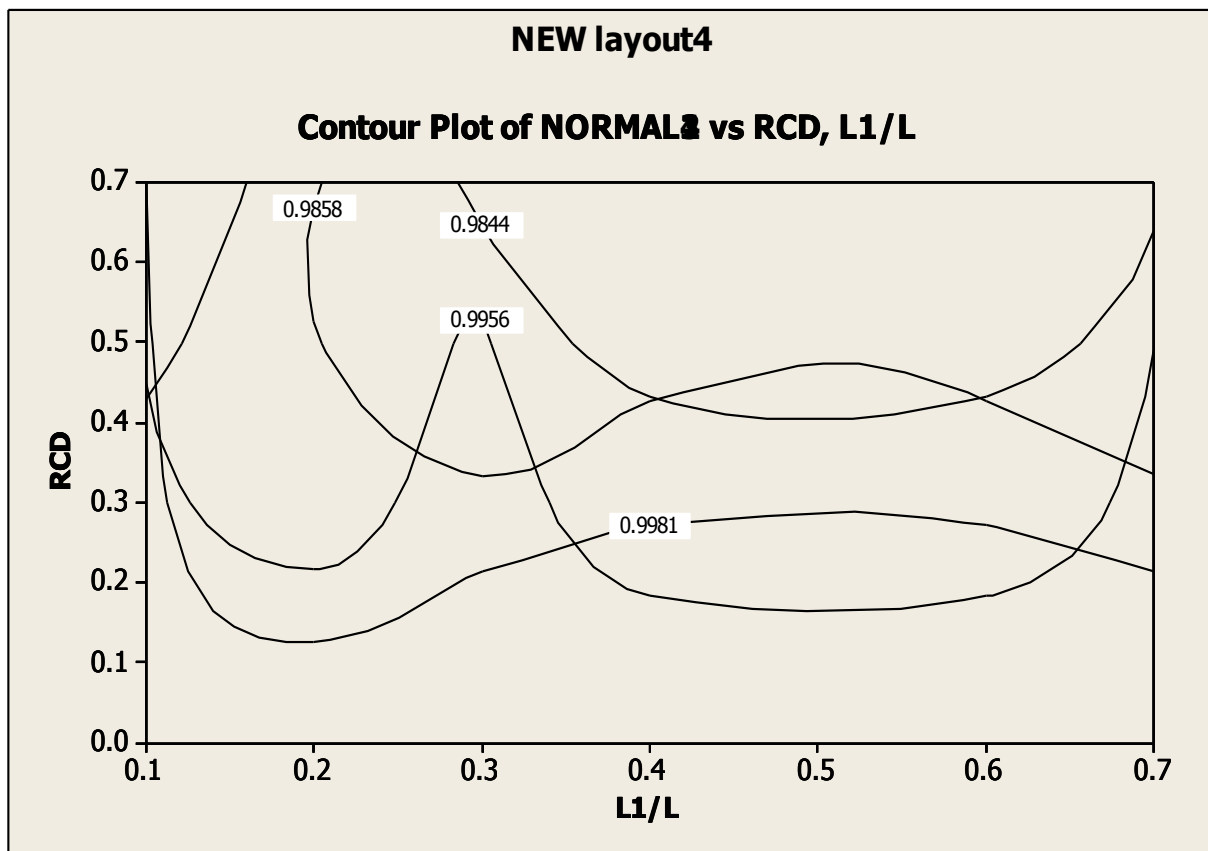


Figure 4.6 Crack depths of 8m at a distance of 275mm

**Table 4.7 Comparison of results**

<b>CRACK PARAMETERS (ACTUAL)</b>	<b>SINHA ET AL ANALYSIS</b>	<b>PRESENT ANALYSIS</b>
CRACK LOCATION = 275mm CRACK DEPTH = 8mm	CRACK LOCATION =299.64mm CRACK DEPTH = 7.082mm	CRACK LOCATION=201.52mm CRACK DEPTH =9mm
CRACK LOCATION = 275mm CRACK DEPTH = 12mm	CRACK LOCATION =274.8mm CRACK DEPTH =11.68mm	CRACK LOCATION =201.52mm CRACK DEPTH =12.75mm

## 4.2 IDENTIFICATION OF CRACK IN ALUMINIUM BEAM

After the comparison the same procedure is used for another aluminum beam with the following parameters:

- Length of the beam = 300mm
- Width of the beam = 9.2mm
- Depth of the beam = 9.2mm
- Young's Modulus of the beam = 68GPa
- Density of the beam =  $2659\text{Kg}/\text{m}^3$
- Poisson's ratio of the beam = 0.33
- No of elements =16
- DOF at each node=2(rotation & translation)
- Element Length =  $\frac{0.3}{16} = 0.01875\text{m}$



The first four modal frequencies for varying crack locations and crack depths are calculated analytically (using the FORTRAN code). Then the frequency contours are plotted for the normalised modal frequency for varying relative crack depth and crack location. The modal experimental normalised frequencies for specimen which the crack location and depth are obtained from PULSE software using FFT analyser. The corresponding modal normalised frequency contours are then retrieved and they are overlapped to get crack depth and crack location of the beam. The results obtained are compared with the actual crack parameters (location and crack depth).

The experimentation included the following apparatus

- Modal hammer(B&K 2302-5)
- Deltatron Accelerometer (B & K4507)
- Portable FFT Analyzer (B & K 3560C)
- Display unit (Desktop)
- Beam specimens(intact and cracked)

Only first three modal frequencies were obtained experimentally in the present case. So the three modal frequency contours are overlapped to get the intersection point. The results of both analytical and experimental are tabulated.

**Table 4.8 Normalized frequency for varying crack depths and locations for first mode**

L1/L	RCD=0.0	RCD=0.1	RCD=0.3	RCD=0.5	RCD=0.7
0.1	1	0.998751	0.985649	0.957565	0.908989
0.2	1	1.000405	0.999919	0.998345	0.993958
0.3	1	0.999845	0.996241	0.986984	0.96336
0.4	1	0.998359	0.985508	0.953477	0.877544
0.6	1	0.998358	0.985507	0.953495	0.877629
0.7	1	0.999861	0.996252	0.986714	0.961842

**Table 4.9 Normalized frequency for varying crack depths and locations for second mode**

L1/L	RCD=0.0	RCD=0.1	RCD=0.3	RCD=0.5	RCD=0.7
0.1	1	1.001204	0.998815	0.9936	0.984503
0.2	1	0.999448	0.991861	0.972796	0.928811
0.3	1	0.997272	0.977094	0.930938	0.841417
0.4	1	0.998985	0.98842	0.963831	0.91482
0.6	1	0.998979	0.988415	0.963937	0.915352
0.7	1	0.997274	0.977095	0.930916	0.84136

**Table 4.10 Normalized frequency for varying crack depths and locations for third mode**

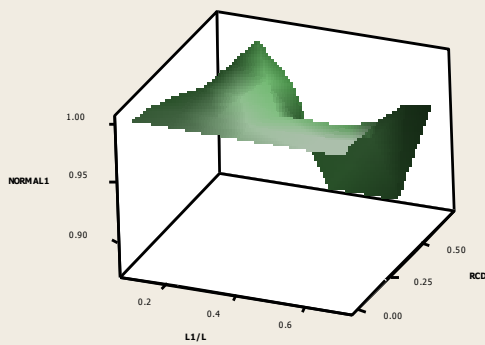
L1/L	RCD=0.0	RCD=0.1	RCD=0.3	RCD=0.5	RCD=0.7
0.1	1	1.002427	1.002183	1.001087	0.997441
0.2	1	0.997425	0.978903	0.938418	0.869172
0.3	1	0.999701	0.992875	0.978516	0.954934
0.4	1	1.000518	0.995588	0.98355	0.958326
0.6	1	1.0005	0.995576	0.983863	0.960007
0.7	1	0.999732	0.992897	0.97802	0.952417

**Table 4.11 Normalized frequency for varying crack depths and locations for fourth mode**

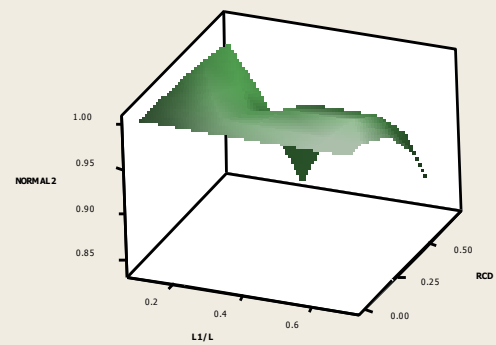
L1/L	RCD=0.0	RCD=0.1	RCD=0.3	RCD=0.5	RCD=0.7
0.1	1	1.002343	0.998618	0.989591	0.970531
0.2	1	0.998995	0.98456	0.956847	0.91771
0.3	1	1.002139	0.999596	0.99359	0.981418
0.4	1	0.997346	0.977722	0.938329	0.87964
0.6	1	0.997343	0.97772	0.938387	0.880056
0.7	1	1.002258	0.999681	0.991368	0.968099

### 3-D PLOT OF NORMALIZED FREQUENCY

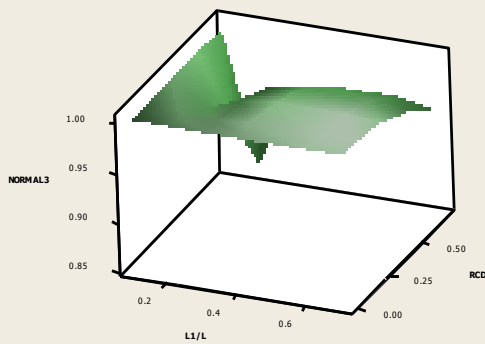
Surface Plot of NORMAL1 vs RCD, L1/L



Surface Plot of NORMAL2 vs RCD, L1/L



Surface Plot of NORMAL3 vs RCD, L1/L



Surface Plot of NORMAL4 vs RCD, L1/L

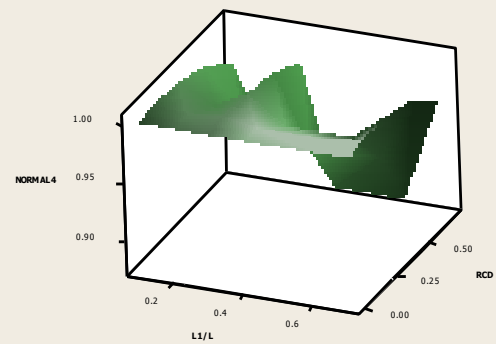


Figure 4.7 Surface plot of normalised modal frequencies

## NORMALIZED FREQUENCY CONTOURS PLOTTED

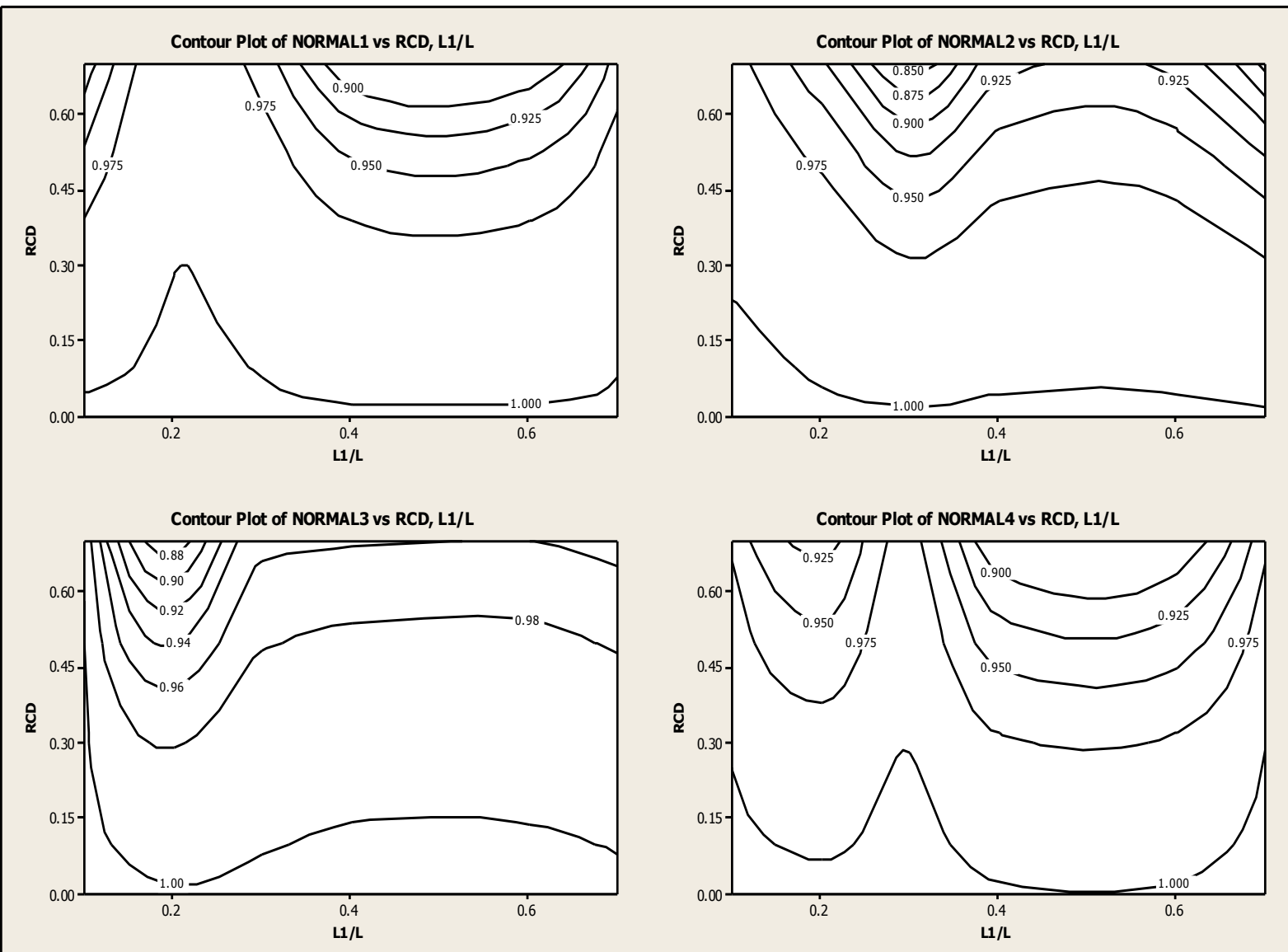


Figure 4.8 Contour plot of noramlised modal frequencies

## EXPERIMENTAL FREQUENCY

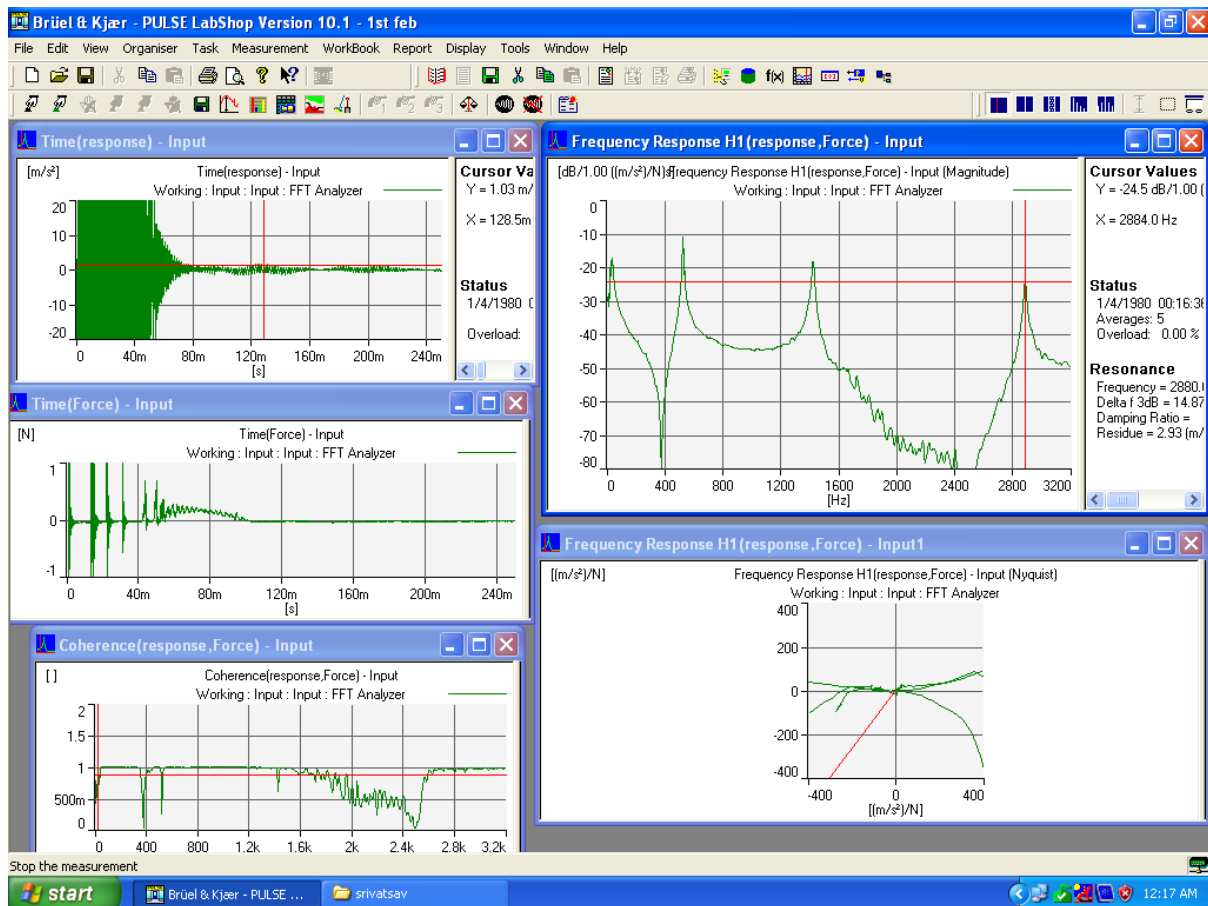


Figure 4.9 Determination of experimental frequencies

Table 4.12 Detemination normalised experimental modal frequencies(rad/s)

MODE	INTACT BEAM FREQUENCY	CRACKED BEAM FREQUENCY	NORMALISED FREQUENCY
MODE 1	3320.2017	3240.48	0.9759
MODE 2	9092.7927	8942.72	0.9835
MODE 3	17679.5869	17546.322	0.9925

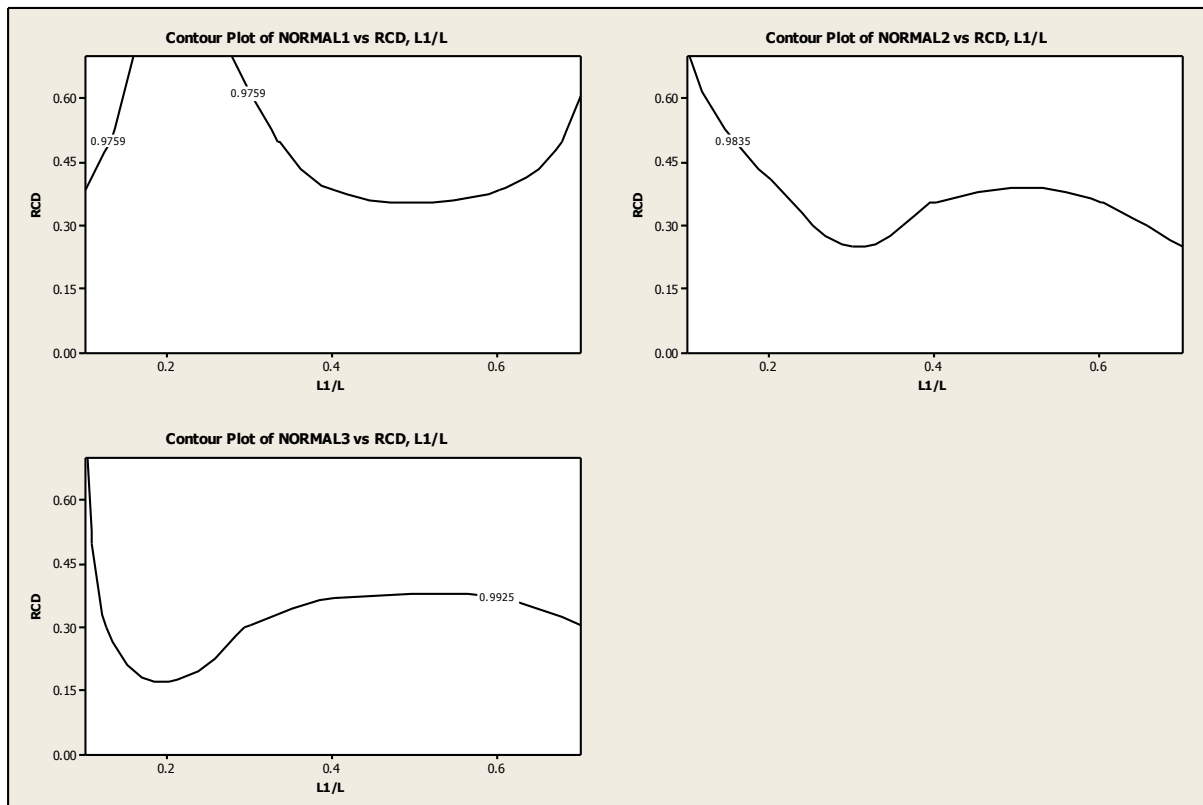


Figure 4.10 Crack normalised frequency retrieved

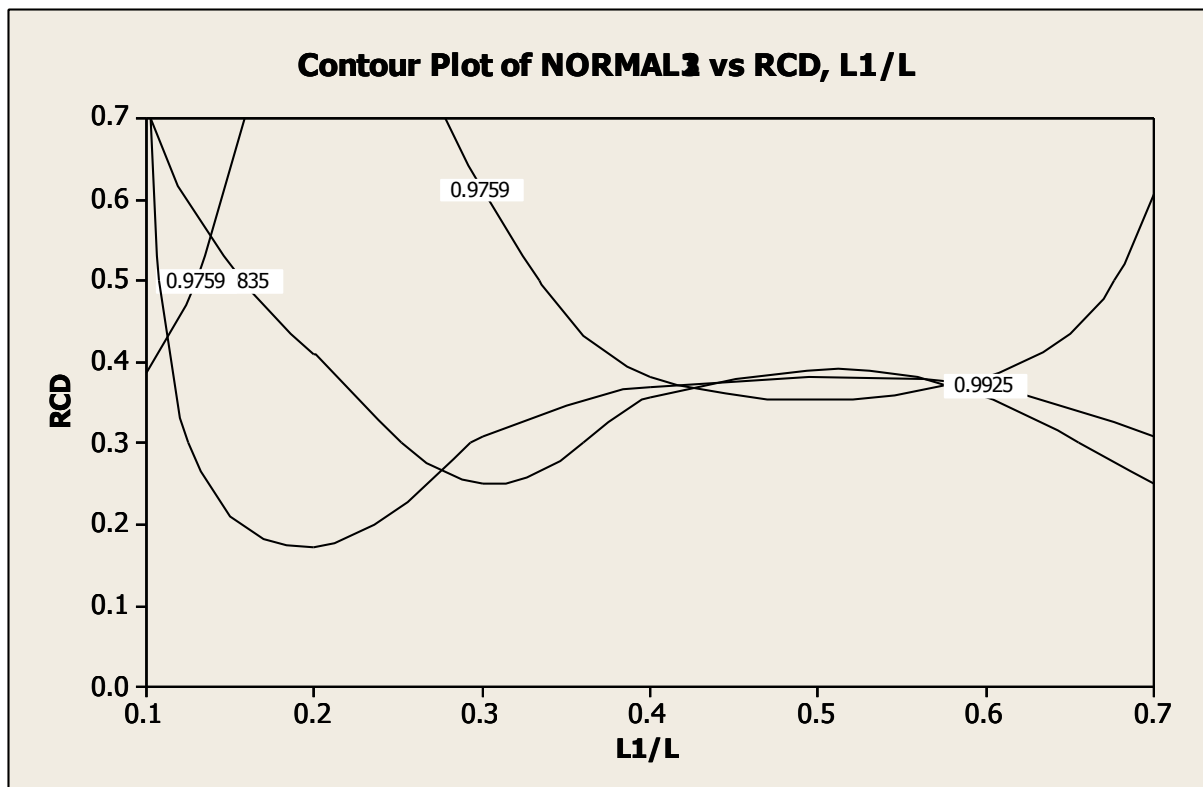


Figure 4.11 Identification of crack location and crack depth

**Table 4.13 Comparison of results with that of actual parameters**

<b>ACTUAL CRACK PARAMETERS</b>	<b>PRESENT ANALYSIS</b>
CRACK LOCATION=10mm	CRACK LOCATION=12.6mm
CRACK DEPTH=3mm	CRACK DEPTH=3.4mm



# CHAPTER-5

## CONCLUSION

## 5.1 CONCLUSION AND FURTHER SCOPE OF STUDY

1. Vibration behavior of beam is very sensitive to crack location, crack depth and mode number. Frequency decreases largely with the increase in crack depth and mode number but in case of crack location it also depends on boundary conditions.
2. The results slightly deviate from the actual parameters due to variation in the analytical and experimental frequencies which are in turn due to the assumptions about damping.
3. It is also seen that error in crack location is more than the crack depth. We are getting more accurate results in the severity cases which are actually more relevant than location as this helps us to decide whether to repair it or not.
4. The study can be extended by incorporating the error i.e by taking the mean of the error between experimental and analytical modal frequency values for varying crack depths and crack locations and thereby reducing it from the analytical value. This will help to get the result in the practical cases and will give the results in the vicinity of the damage. These will greatly reduce the labour, time and cost making it effective in use.
5. The Cantilever and Simply Supported cracked beams can also be analysed using this method. But in case of Cantilever and Simply supported beams the error between the experimental results and analytical results will be more due to variation of boundary stiffnesses between analytical and experimental, so error incorporation gives better results.
6. This method can be extended to beams with multiple cracks.

# CHAPTER-6

## REFERENCES

## **REFERENCES**

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